Student Name: ____________________ Student Number: ______________

Part I:

**Question 1: (9 Marks:)**
**Evaluate the following limits:**

(a) $\lim_{x \to 0} \frac{\sin 2x}{2x}$

**(b)** $\lim_{x \to 0} \frac{x^2 - 1}{x^2 + x}$

**(c)** $\lim_{x \to 0} \frac{x}{\sqrt{1 + 4x} - 1}$

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Question 2: (20 Marks)

Find $\frac{dy}{dx}$ for each of the following: (Do not simplify)

(a) $y = 3x^4 - \frac{5}{x^3} + 7\sqrt{x} - \pi^2$

(b) $y = \sqrt{\arcsin x}$ (Hint: $\arcsin x = \sin^{-1} x$ )

(c) $y = \frac{xe^{-x^3}}{x^2+1}$

(d) $y = \sin^2(\tan x^3)$

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Question 3: (5 Marks)

(a) Find the equation of the tangent line to the graph of $y = \sin x$ at the point $(\pi, 0)$.

(b) Use implicit differentiation to find $\frac{dy}{dx}$ for $(x^2 + y^2)^2 = 2xy$

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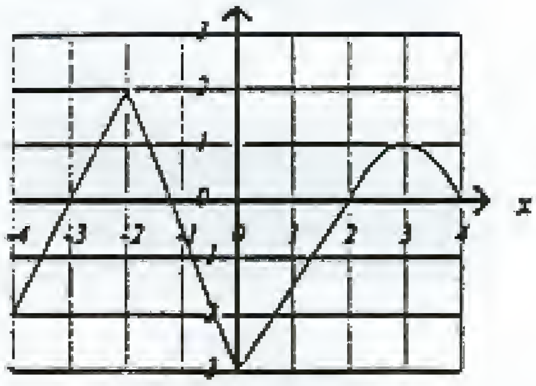
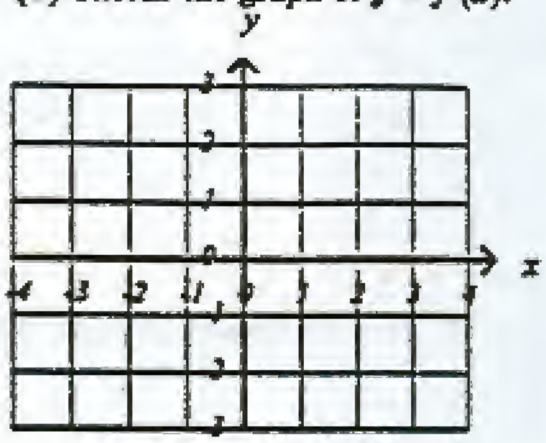
Question 4: (12 Marks)
The graph of $y = f(x)$ is shown below. It is made up of three straight line segments, and part of a parabola whose equation is $y = 1 - (x-3)^2$.

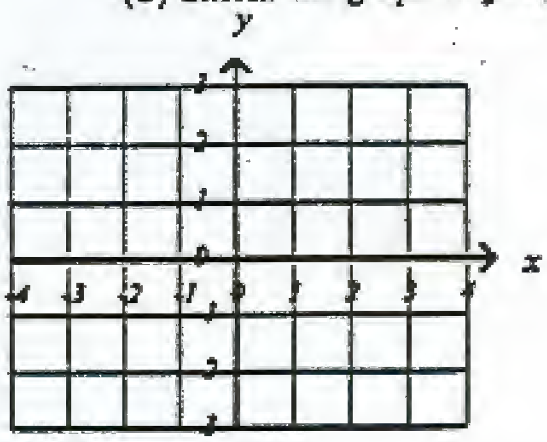



$y = f(x)$

**(a) Sketch the graph of $y = f'(x)$.**




**(b) Sketch the graph of $y = f''(x)$.**




**(c) Find $f'(f(1))$.**

**(d) Evaluate $\int_{-4}^{2} f(x)dx$.**

**(e) Find the area above the $x$-axis and below $y = f(x)$ between $x = 2$ and $x = 4$. (Hint: express it as a definite integral.)**

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**Question 5: (5 Marks)**

(a) Find general antiderivatives $F(x)$ of the function $f(x)$ if :

i) $f(x) = 3\sqrt{x}$

ii) $f(x) = -\frac{1}{x}$

iii) $f(x) = e^x$

(b) Determine the antiderivative $F(x)$ of the function $f(x) = 3\sqrt{x} - \frac{1}{x} + e^x$ for w
$F(1) = 1 - e$.

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**Question 6:** (12 Marks)
Consider the function $f(x) = -x^4 + 4x^3 - 3$.

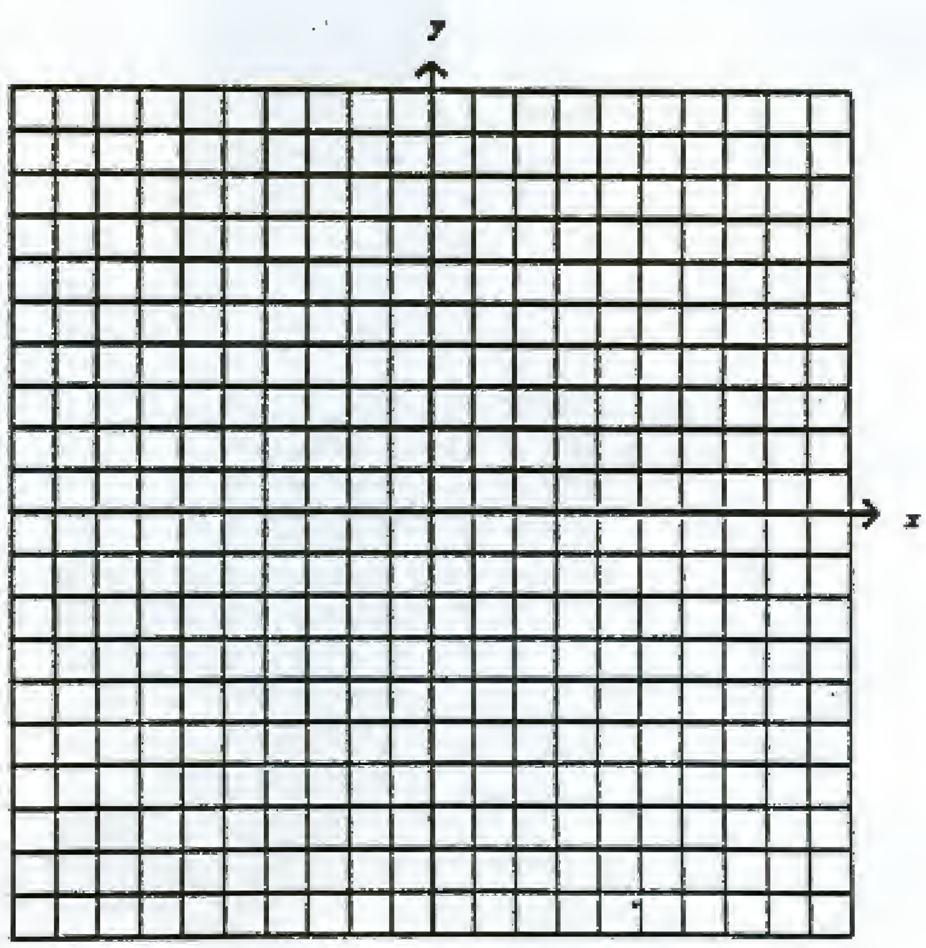
(a) find the intervals where $f$ is increasing and where $f$ is decreasing

(b) find any local maxima or minima

(c) find the intervals where $f$ is concave up and where $f$ is concave down

(d) find any points of inflection

(e) sketch a graph of $y = f(x)$



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**Question 7: (5 Marks)**
Which values of $a$ and $b$ will make the function

$$f(x) = \begin{cases} x + a & \text{for } x < 0 \\ x^2 + bx + 2 & \text{for } x \geq 0 \end{cases}$$

i) continuous on $(-\infty, \infty)$?

ii) differentiable on $(-\infty, \infty)$?

**Question 8: (5 Marks)** A certain function $N(t)$ satisfies the exponential growth law $N(3) = 3000$ and $N(6) = 6000$, what is $N(4)$?

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**Question 9: (6 Marks)**

Consider the function $f(x) = \dfrac{1}{x^2 - 4}$

(a) determine $\lim_{x \to \infty} f(x)$, $\lim_{x \to -\infty} f(x)$, and any horizontal asymptotes of the graph of $y = f(x)$

(b) determine any vertical asymptotes of the graph of $y = f(x)$ and for each such asymptote $x = a$ determine $\lim_{x \to a^+} f(x)$ and $\lim_{x \to a^-} f(x)$.

**Question 10: (5 Marks)**

**(a) Explain the meaning of** $\displaystyle\sum_{i=2}^{4} \frac{1}{\sin\frac{\pi}{i}}$**, i.e. expand the sum.**

**(b) Evaluate it correctly to three decimal places.**

**Remember:** $\sqrt{2} \doteq 1.4142$, $\sqrt{3} \doteq 1.7321$.

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**Part II:**

Do no more than two of the following three questions, each of which is worth 10 marks:

**Question 11: (8 Marks)**

Sand is being dumped on a pile in such a way that it always forms a cone whose ra
equals its height. If the sand is being dumped at a rate of 10 cubic feet per minute, at w
rate is the height of the pile increasing when there is $\frac{1000}{3\sqrt{\pi}}$ cubic feet of sand in the pile?

Remember:The volume $V$ of a cone of height $h$ and radius $r$ is $\frac{\pi}{3}r^2h$

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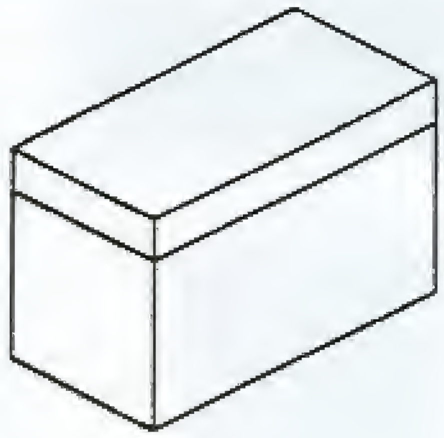
**Question 12: (8 Marks)**
A three litre, two-piece rectangular metal cookie container is to be constructed so that the length of its base is $\frac{5}{3}$ times its width. The lid is to overlap the bottom (cookie bearing) of the container by one-fifth of the height of the container. Find the dimensions that will use the least amount of metal, assuming that the thickness of the metal sheet used to make it is zero. Remember that one litre is equal to one thousand cubic centimetres.



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Question 13: (8 Marks)

Oil is pumped continuously from a well at a rate proportional to the amount of oil left in well. Initially there were 1,000,000 barrels of oil in the well; 6 years later, 500,000 ba remain.

(a) Give a formula for the amount of oil, $B(t)$, in the well (in barrels) as a functio time $t$ (in years).

(b) At what rate was the amount of oil in the well decreasing when there were 60( barrels of oil remaining?

(c) It will no longer be profitable to pump oil from the well when there are fewer than 5 barrels remaining. The plan is to pump oil for 24 years, will the well be profitable fo length of time?

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